University of South Carolina
MATH 544-001

Midterm Examination 1 - Sample B

September 12, 2024

Closed book examination | Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find the reduced row echelon form for the matrix

$$\begin{pmatrix} 1 & 3 & -2 & 4 \\ 1 & 3 & -2 & 4 \\ 2 & 3 & 1 & 0 \\ -2 & 3 & 0 & 4 \end{pmatrix}.$$

Problem 2. (5 points) Describe all solutions of the following linear system.

$$\begin{aligned} x_1 + 2x_3 - 4x_4 + 7x_6 &= 5 \\ x_2 + 3x_3 + 5x_4 + 6x_6 &= 2 \\ x_5 - 9x_6 &= 2 \end{aligned}$$

Problem 3. (5 points) Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric vector form, where

$$A = \begin{pmatrix} 1 & 2 & 0 & 0 & 3 \\ 0 & 0 & 1 & 0 & 4 \\ 0 & 0 & 0 & 1 & 5 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 6 \\ 7 \\ 8 \end{pmatrix}.$$

Problem 4. (5 points) Determine if the columns of the matrix

$$\begin{pmatrix} 3 & 9 & 3 & 7 \\ 1 & 3 & 1 & 5 \\ 1 & 3 & 2 & 0 \\ 2 & 6 & 0 & 4 \end{pmatrix}$$

form a linearly independent set.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 5. (1 point) Every elementary row operation is reversible.

Problem 6. (1 point) When $\mathbf{u}$ and $\mathbf{v}$ are nonzero vectors, $\text{Span}\{\mathbf{u}, \mathbf{v}\}$ contains the line through $\mathbf{u}$ and the origin.

Problem 7. (1 point) Any finite linear combination of vectors can always be written in the form $A\mathbf{x}$ for a suitable matrix A and vector $\mathbf{x}$.

Problem 8. (1 point) A linear system is homogeneous if and only if it has a unique solution.

Problem 9. (1 point) If a vector equation has a solution, then the zero vector is a solution

Problem 10. (5 points) Let S be the set of vectors in $\mathbb{R}^n$ with exactly two entries equal to one and the rest zero. Show that if $n > 3$, then S is linearly dependent.

The End