University of South Carolina

MATH 544-001

Midterm Examination 1

September 12, 2024

Closed book examination Time: 75 minutes

Instructions:

Notes, books, computer, phones, calculators or other aids are **not** allowed. Please write on only one side of each page. If you need more space than is provided, then ask for extra paper from the proctor. Simplify your final answers. Full credit will not be awarded for insufficient accompanying work.

There are 30 total points available.

Problem 1. (5 points) Find the reduced row echelon form for the matrix

$$\begin{pmatrix} 1 & 0 & 2 & 3 \\ 2 & 2 & 0 & 5 \\ 3 & 2 & 1 & 4 \end{pmatrix}.$$

Problem 2. (5 points) Describe all solutions of the following linear system.

$$\begin{aligned} x + 2y + 2w - v &= 1 \\ z + 3w + 5v &= 2 \\ u + v &= 7 \end{aligned}$$

Problem 3. (5 points) Describe all solutions of $A\mathbf{x} = \mathbf{b}$ in parametric vector form, where

$$A = \begin{pmatrix} 1 & 0 & 2 & 4 & 0 & 4 \\ 0 & 1 & 3 & 5 & 0 & 2 \\ 0 & 0 & 0 & 0 & 1 & 7 \end{pmatrix} \text{ and } \mathbf{b} = \begin{pmatrix} 5 \\ 3 \\ 2 \end{pmatrix}.$$

Problem 4. (5 points) Determine if the vectors

$$\begin{pmatrix}1\\2\\4\end{pmatrix}, \quad \begin{pmatrix}1\\3\\5\end{pmatrix}, \text{ and } \begin{pmatrix}1\\4\\6\end{pmatrix}.$$

are linearly independent.

Determine whether each of the following statements are true or false. No justification is necessary.

Problem 5. (1 point) Every linear system has a unique solution.

Problem 6. (1 point) If A is an $n \times m$ matrix and $\mathbf{u}$ and $\mathbf{v}$ are vectors in $\mathbb{R}^m$, then $A(\mathbf{u}+\mathbf{v}) = A\mathbf{u}+A\mathbf{v}$.

Problem 7. (1 point) The vectors $\mathbf{v}_1, \ldots, \mathbf{v}_p$ in $\mathbb{R}^n$ are linearly dependent if and only if the equation $c_1\mathbf{v}_1 + \cdots + c_p\mathbf{v}_p = \mathbf{0}$ has a solution.

Problem 8. (1 point) If the equation $A\mathbf{x} = \mathbf{b}$ is inconsistent, then $\mathbf{b}$ is not in the set spanned by the columns of A.

Problem 9. (1 point) Whenever a system has free variables, the solution set contains a unique solution.

Problem 10. (5 points) Let $\mathbf{u}, \mathbf{v}, \mathbf{w}$ be vectors in $\mathbb{R}^n$. Show that, if $\mathbf{w}$ is in the set spanned by $\mathbf{v}$ and $\mathbf{u}$, then $\mathbf{u}, \mathbf{v}, \mathbf{w}$ are linearly dependent.

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