Problem 1 Rewrite the polynomial

$$x^4 + 2x^3y^3z + 3x^4yz^2 + 4x^3y^2z^2$$

using each of the *lex*, *grlex*, and *grevlex* orders.

Problem 2 Compute the quotient and remainder of $2x^4 - 3x^2 + 1$ on division by $x^2 - 3x + 1$.

Problem 3 Consider the following polynomials:

$$f_1 = x^4 + x^3 - x - 1$$
$$f_2 = x^4 + x^3 + 3x^2 + 2x + 2$$
$$f_3 = x^4 + x^2 - 2$$

Compute $\gcd(f_1, f_2, f_3)$.

Problem 4 Consider the following polynomials:

$$f = x^4 + 3x^2 + 2 \qquad \begin{aligned} g_1 &= x^3 - 3x^2 + 2x - 6 \\ g_2 &= x^3 - 2x^2 + 2x - 4 \end{aligned}$$

Is f in the ideal $\langle g_1, g_2 \rangle$?

Problem 5 Consider the following polynomials:

$$f = x^3y^2 - xy^3 + 3xy \qquad \begin{aligned} g_1 &= xy^2 + 2y \\ g_2 &= x^2y - xy \\ g_3 &= 3y^2 - 4 \end{aligned}$$

Compute the remainder of f on division by the ordered set $G = (g_1,\ g_2,\ g_3)$ using the division algorithm and the *lex* order.

Problem 6 Find the minimal basis for the following monomial ideal:

$$\langle x^3yz,\ xy^2z,\ xy^3z^2,\ x^3y^2z^2,\ x^4,\ y^4,\ z^3,\ x^2yz^4 \rangle$$

Problem 7 Suppose $f_1, \ldots, f_s, g_1, \ldots, g_t$ are polynomials in $k[x_1, \ldots, x_n]$ where k is a field. Prove or disprove each of the following:

(1) If $\mathbf{V}(f_1, \ldots, f_s) = \mathbf{V}(g_1, \ldots, g_t)$, then $\langle f_1, \ldots, f_s \rangle = \langle g_1, \ldots, g_t \rangle$.
(2) If $\langle f_1, \ldots, f_s \rangle \subseteq \langle g_1, \ldots, g_t \rangle$, then $\mathbf{V}(f_1, \ldots, f_s) \subseteq \mathbf{V}(g_1, \ldots, g_t)$.
(3) If $\langle f_1, \ldots, f_s \rangle \subseteq \langle g_1, \ldots, g_t \rangle$, then $\mathbf{V}(g_1, \ldots, g_t) \subseteq \mathbf{V}(f_1, \ldots, f_s)$.

Problem 8 For $\alpha = (\alpha_1, \ldots, \alpha_n), \beta = (\beta_1, \ldots, \beta_n) \in \mathbb{N}^n$, define $\alpha >_{max} \beta$ if and only if

- $\max\{\alpha_1, \ldots, \alpha_n\} > \max\{\beta_1, \ldots, \beta_n\}$, or
- $\max\{\alpha_1, \ldots, \alpha_n\} = \max\{\beta_1, \ldots, \beta_n\}$ and $\alpha >_{lex} \beta$.

Determine if $>_{max}$ is a monomial order. Prove your answer either way.